Inner–Outer Decomposition for Constrained Finite-Horizon LQ Differential Games

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Abstract

We study a finite-horizon constrained linear-quadratic robust control problem, formulated as a differential game between a control player and and a disturbance player (the nature). The system dynamics are given by

$$\dot{x}(t) = Ax(t) + B_u u(t) + B_w w(t), \qquad x(t_0) = x_0,$$

where the control is subject to pointwise constraints $u(t) \in U$. The disturbance is treated in a worst-case sense, and the objective is to design a control that guarantees performance against all admissible disturbances. This problem is motivated by the infinite-horizon robust control setting for which finite-horizon differential games arise as natural approximations.

Our approach is based on an inner-outer decomposition. For a fixed control, the inner maximization problem with respect to the disturbance admits an explicit solution via a Riccati equation, leading to a reduced optimization problem involving only the constrained control. The latter is addressed by a penalized Sakawa-Shindo-type algorithm [1], based on pointwise minimization over the admissible set.

The main contribution is a convergence analysis of the sequence of controls generated by the algorithm. We establish, in particular, a descent property for the reduced functional and the existence of a point-wise convergent subsequence of controls. The resulting framework provides a constructive and analytically justified approach to constrained finite-horizon linear-quadratic differential games, as well as a method for approximating infinite-horizon robust control problems.

References

[1] N. Sakawa and Y. Shindo, *On the solution of optimal control problems with bounded controls*, SIAM Journal on Control, 1979.